

Axino Dark Matter in Anomalous $U(1)'$ Models

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Abstract

We study a possible dark matter candidate in the framework of a minimal anomalous $U(1)'$ extension of the MSSM. It turns out that in a suitable decoupling limit the axino, which is present in the Stüeckelberg multiplet, is the lightest supersymmetric particle (LSP). We compute the relic density of this particle including coannihilations with the next to lightest supersymmetric particle (NLSP) and with the next to next to lightest supersymmetric particle (NNLSP) which are assumed almost degenerate in mass. This assumption is needed in order to satisfy the stringent limits that the Wilkinson Microwave Anisotropy Probe (WMAP) puts on the relic density. We find that the WMAP constraints can be satisfied by different NLSP and NNLSP configurations as a function of the mass gap with the LSP. These results hold in the parameter space region where the model remains perturbative.

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1 Introduction

A great deal of work has been done recently to embed the standard model of particle physics (SM) into a brane construction [1, 2, 3, 4]. This research is part of the effort, initiated in [5], to build a fully realistic four dimensional vacuum out of string theory. While the original models were formulated in the framework of the heterotic string, the most recent efforts were formulated for type II strings in order to take advantage of the recent work on moduli stabilization using fluxes. Such brane constructions naturally lead to extra anomalous $U(1)$'s in the four dimensional low energy theory and, in turn, to the presence of possible heavy Z' particles in the spectrum. These particles should be among the early findings of LHC and besides for the above cited models they are also a prediction of many other theoretical models of the unification of forces (see [6] for a recent review). It is then of some interest to know if these Z' particles contribute to the cancellation of the gauge anomaly in the way predicted from string theory or not. In [7] some of the present authors have studied a supersymmetric (SUSY) extension of the minimal supersymmetric standard model (MSSM) in which the anomaly is cancelled *à la* Green-Schwarz. The model is only string-inspired and is not the low-energy sector of some brane construction. The reason of this choice rests in our curiosity to explore the phenomenology of these models keeping a high degree of flexibility, while avoiding the intricacies and uncertainties connected with a string theory construction. For previous work along these lines we refer to [8, 9, 10, 11, 12, 13]. In this work we perform a consistency check of our model [7] by evaluating the thermal relic density of the model to compare it against the WMAP data. If the axino is the lightest supersymmetric particle (LSP), its relic density is too high with respect to the experimental data. This is why we favor a next to lightest supersymmetric particle (NLSP) with a mass close to the LSP. We show that an interesting scenario arise also for three particle coannihilation processes. In these two cases, the model is consistent with the experimental data. Moreover in the three particle case we find configurations in which the LSP and the NLSP do not need to be nearly degenerate in mass. In this case the mass gap between the two can be of the order of 20%. This is the plan of the paper: in Section 2 we describe our model. In Section 3 and 4 we find the LSP and study the axino interactions. Finally in Section 5 we compute the relic density. Section 6 is a summary of our results.

2 Model Setup

In this section we briefly discuss our theoretical framework. We assume an extension of the MSSM with an additional abelian vector multiplet $V^{(0)}$ with arbitrary charges. The

anomalies are cancelled with the Green-Schwarz (GS) mechanism and with the Generalized Chern-Simons (GCS) terms. All the details can be found in [7]. All the MSSM fields are charged under the additional vector multiplet $V^{(0)}$, with charges which are given in Table 1, where Q_i, L_i are the left handed quarks and leptons respectively while U_i^c, D_i^c, E_i^c are the right handed up and down quarks and the electrically charged leptons. The superscript c stands for charge conjugation. The index $i = 1, 2, 3$ denotes the three different families. $H_{u,d}$ are the two Higgs scalars.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
Q_i	3	2	1/6	Q_Q
U_i^c	$\bar{\mathbf{3}}$	1	-2/3	Q_{U^c}
D_i^c	$\bar{\mathbf{3}}$	1	1/3	Q_{D^c}
L_i	1	2	-1/2	Q_L
E_i^c	1	1	1	Q_{E^c}
H_u	1	2	1/2	Q_{H_u}
H_d	1	2	-1/2	Q_{H_d}

Table 1: Charge assignment.

The key feature of this model is the mechanism of anomaly cancellation. As it is well known, the MSSM is anomaly free. In our MSSM extension all the anomalies that involve only the $SU(3)$, $SU(2)$ and $U(1)_Y$ factors vanish identically. However, triangles with $U(1)'$ in the external legs in general are potentially anomalous. These anomalies are⁵

$$U(1)' - U(1)' - U(1)' : \quad \mathcal{A}^{(0)} \quad (1)$$

$$U(1)' - U(1)_Y - U(1)_Y : \quad \mathcal{A}^{(1)} \quad (2)$$

$$U(1)' - SU(2) - SU(2) : \quad \mathcal{A}^{(2)} \quad (3)$$

$$U(1)' - SU(3) - SU(3) : \quad \mathcal{A}^{(3)} \quad (4)$$

$$U(1)' - U(1)' - U(1)_Y : \quad \mathcal{A}^{(4)} \quad (5)$$

All the remaining anomalies that involve $U(1)'$'s vanish identically due to group theoretical arguments (see Chapter 22 of [14]). Consistency of the model is achieved by the contribution of a Stückelberg field S and its appropriate couplings to the anomalous $U(1)'$. The Stückelberg lagrangian written in terms of superfields is [15]

$$\mathcal{L}_S = \frac{1}{4} (S + S^\dagger + 4b_3 V^{(0)})^2 \Big|_{\theta^2 \bar{\theta}^2} - \frac{1}{2} \left\{ \left[\sum_{a=0}^3 b_2^{(a)} S \text{Tr} (W^{(a)} W^{(a)}) + b_2^{(4)} S W^{(1)} W^{(0)} \right] + h.c. \right\} \Big|_{\theta^2} \quad (6)$$

⁵We are working in an effective field theory framework and we ignore throughout the paper all the gravitational effects. In particular, we do not consider the gravitational anomalies which, however, could be canceled by the Green-Schwarz mechanism.

where the index $a = 0, \dots, 3$ runs over the $U(1)'$, $U(1)_Y$, $SU(2)$ and $SU(3)$ gauge groups respectively. The Stückelberg multiplet is a chiral superfield

$$S = s + i\sqrt{2}\theta\psi_S + \theta^2 F_S - i\theta\sigma^\mu\bar{\theta}\partial_\mu s + \frac{\sqrt{2}}{2}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi_S - \frac{1}{4}\theta^2\bar{\theta}^2\Box s \quad (7)$$

and transforms under $U(1)'$ as

$$\begin{aligned} V^{(0)} &\rightarrow V^{(0)} + i(\Lambda - \Lambda^\dagger) \\ S &\rightarrow S - 4i b_3 \Lambda \end{aligned} \quad (8)$$

where b_3 is a constant related to the Z' mass. We give the expansion of \mathcal{L}_S in component fields only for the part that is needed in the following sections. Using the Wess-Zumino gauge we get

$$\begin{aligned} \mathcal{L}_{axino} &= \frac{i}{4}\psi_S\sigma^\mu\partial_\mu\bar{\psi}_S - \sqrt{2}b_3\psi_S\lambda^{(0)} - \frac{i}{2\sqrt{2}}\sum_{a=0}^2 b_2^{(a)}\text{Tr}(\lambda^{(a)}\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu}^{(a)})\psi_S \\ &\quad - \frac{i}{2\sqrt{2}}b_2^{(4)}\left[\frac{1}{2}\lambda^{(1)}\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu}^{(0)}\psi_S + (0 \leftrightarrow 1)\right] + h.c. \end{aligned} \quad (9)$$

As it was pointed out in [8], the Stückelberg mechanism is not enough to cancel all the anomalies. Mixed anomalies between anomalous and non-anomalous factors require an additional mechanism to ensure consistency of the model: non gauge invariant GCS terms must be added. In our case, the GCS terms have the form [16]

$$\begin{aligned} \mathcal{L}_{GCS} &= -d_4 \left[(V^{(1)}D^\alpha V^{(0)} - V^{(0)}D^\alpha V^{(1)}) W_\alpha^{(0)} + h.c. \right]_{\theta^2\bar{\theta}^2} + \\ &\quad + d_5 \left[(V^{(1)}D^\alpha V^{(0)} - V^{(0)}D^\alpha V^{(1)}) W_\alpha^{(1)} + h.c. \right]_{\theta^2\bar{\theta}^2} + \\ &\quad + d_6 \text{Tr} \left[(V^{(2)}D^\alpha V^{(0)} - V^{(0)}D^\alpha V^{(2)}) W_\alpha^{(2)} + n.a.c + h.c. \right]_{\theta^2\bar{\theta}^2} \end{aligned} \quad (10)$$

where $n.a.c.$ refers to non abelian completion terms. The b constants in (6) and the d constants in (10) are fixed by the anomaly cancellation procedure (for details see [7]).

For a symmetric distribution of the anomaly, we have

$$\begin{aligned} b_2^{(0)}b_3 &= -\frac{\mathcal{A}^{(0)}}{384\pi^2} & b_2^{(1)}b_3 &= -\frac{\mathcal{A}^{(1)}}{128\pi^2} & b_2^{(2)}b_3 &= -\frac{\mathcal{A}^{(2)}}{64\pi^2} & b_2^{(4)}b_3 &= -\frac{\mathcal{A}^{(4)}}{128\pi^2} \\ d_4 &= -\frac{\mathcal{A}^{(4)}}{384\pi^2} & d_5 &= \frac{\mathcal{A}^{(1)}}{192\pi^2} & d_6 &= \frac{\mathcal{A}^{(2)}}{96\pi^2} \end{aligned} \quad (11)$$

It is worth noting that the GCS coefficients $d_{4,5,6}$ are fully determined in terms of the \mathcal{A} 's by the gauge invariance, while the $b_2^{(a)}$'s depend only on the free parameter b_3 , which is related to the mass of the anomalous $U(1)$.

The soft breaking sector of the model is given by

$$\mathcal{L}_{soft} = \mathcal{L}_{soft}^{MSSM} - \frac{1}{2}(M_0\lambda^{(0)}\lambda^{(0)} + h.c.) - \frac{1}{2}\left(\frac{M_S}{2}\psi_S\psi_S + h.c.\right) \quad (12)$$

where $\mathcal{L}_{soft}^{MSSM}$ is the usual soft susy breaking lagrangian while $\lambda^{(0)}$ is the gaugino of the added $U(1)'$ and ψ_S is the axino. The axino soft mass term deserves some comment. Based on [17] one could naively expect that such a term is not allowed since the Stückelberg multiplet is a chiral multiplet. This turns out not to be correct because the Stückelberg multiplet couplings are not of the Yukawa type. In fact in our model both the axino and the axion couple only through GS interactions. It is worth noting that a mass term for the axion ϕ is instead not allowed since it transforms non trivially under the anomalous $U(1)'$ gauge transformation (8).

3 Neutralino Sector

Assuming the conservation of R-parity the LSP is a good weak interacting massive particle (WIMP) dark matter candidate. As in the MSSM the LSP is given by a linear combination of fields in the neutralino sector. The general form of the neutralino mass matrix is given in [7]. Written in the interaction eigenstate basis $(\psi^0)^T = (\psi_S, \lambda^{(0)}, \lambda^{(1)}, \lambda_3^{(2)}, \tilde{h}_d^0, \tilde{h}_u^0)$ it is a six-by-six matrix. From the point of view of the strength of the interactions the two extra states are not on the same footing with respect to the standard ones. The axino and the extra gaugino $\lambda^{(0)}$ dubbed primeino are in fact extremely weak interacting massive particle (XWIMP). Thus we are interested in situations in which the extremely weak sector is decoupled from the standard one and the LSP belongs to this sector. This can be achieved at tree level with the choice

$$Q_{H_u} = Q_{H_d} = 0 \quad (13)$$

The neutralino mass matrix $\mathbf{M}_{\tilde{N}}$ becomes

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} \frac{M_S}{2} & \frac{M_{V^{(0)}}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ \dots & M_0 & 0 & 0 & 0 & 0 \\ \dots & \dots & M_1 & 0 & -\frac{g_1 v_d}{2} & \frac{g_1 v_u}{2} \\ \dots & \dots & \dots & M_2 & \frac{g_2 v_d}{2} & -\frac{g_2 v_u}{2} \\ \dots & \dots & \dots & \dots & 0 & -\mu \\ \dots & \dots & \dots & \dots & \dots & 0 \end{pmatrix} \quad (14)$$

where M_S , M_0 , M_1 , M_2 are the soft masses coming from the soft breaking terms (12) while $M_{V^{(0)}}$ is given in eq. (17). It is worth noting that the D terms and kinetic mixing terms can be neglected in the tree-level computations of the eigenvalues and eigenstates.

Moreover we make the assumption that

$$M_0 \gg M_S, M_{V^{(0)}} \quad (15)$$

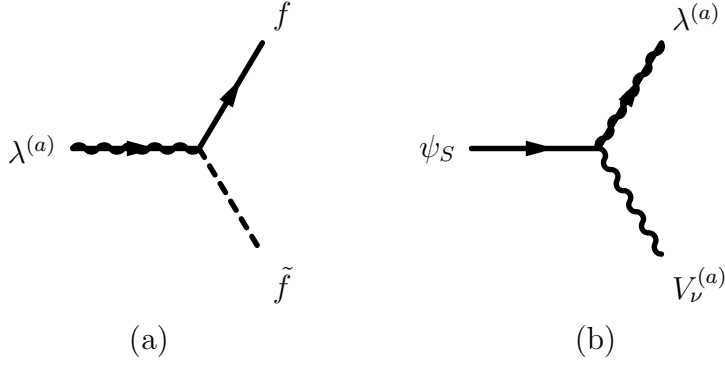


Figure 1: (a) Gaugino-fermion-sfermion interaction vertex. (b) Axino-gaugino-vector interaction vertex.

so that the axino is the LSP. This assumption is motivated by the interaction strengths of the two extra states. Since the axino interacts via the vertex shown in fig. 1b (which is of order $\sim g_0 g_a^2 / M_{Z'}$) while the primeino interacts via the vertex in fig. 1a (which is of order $\sim g_a$), if we do not assume the two decoupling relations (13) and (15) the dominant contribution in the (co)annihilation processes is that of the primeino, which is of the type of a standard gaugino interaction.

The decoupling in the neutralino sector implies also a decoupling in the gauge bosons sector. In our model there are two mechanisms that give mass to the gauge bosons: (i) the Stückelberg mechanism and (ii) the Higgs mechanism. In this extension of the MSSM, the mass terms for the gauge fields for $Q_{H_u} = 0$ are given by

$$\mathcal{L}_M = \frac{1}{2} \begin{pmatrix} V_\mu^{(0)} & V_\mu^{(1)} & V_{3\mu}^{(2)} \end{pmatrix} M^2 \begin{pmatrix} V^{(0)\mu} \\ V^{(1)\mu} \\ V_3^{(2)\mu} \end{pmatrix} \quad (16)$$

with M^2 being the gauge boson mass matrix

$$M^2 = \begin{pmatrix} M_{V^{(0)}} & 0 & 0 \\ \dots & g_1^2 \frac{v^2}{4} & -g_1 g_2 \frac{v^2}{4} \\ \dots & \dots & g_2^2 \frac{v^2}{4} \end{pmatrix} \quad (17)$$

where $M_{V^{(0)}} = 4b_3 g_0$ is the mass parameter for the anomalous $U(1)$ and it is assumed to be in the TeV range. The lower dots denote the obvious terms under symmetrization. After diagonalization, we obtain the eigenstates

$$A_\mu = \frac{g_2 V_\mu^{(1)} + g_1 V_{3\mu}^{(2)}}{\sqrt{g_1^2 + g_2^2}} \quad (18)$$

$$Z_{0\mu} = \frac{g_2 V_{3\mu}^{(2)} - g_1 V_\mu^{(1)}}{\sqrt{g_1^2 + g_2^2}} \quad (19)$$

$$Z'_\mu = V_\mu^{(0)} \quad (20)$$

and the corresponding masses

$$M_\gamma^2 = 0 \quad (21)$$

$$M_{Z_0}^2 = \frac{1}{4} (g_1^2 + g_2^2) v^2 \quad (22)$$

$$M_{Z'}^2 = M_{V_{(0)}}^2 \quad (23)$$

Finally the rotation matrix from the hypercharge to the photon basis is

$$\begin{aligned} \begin{pmatrix} Z'_\mu \\ Z_{0\mu} \\ A_\mu \end{pmatrix} &= O_{ij} \begin{pmatrix} V_\mu^{(0)} \\ V_\mu^{(1)} \\ V_{3\mu}^{(2)} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{g_1}{\sqrt{g_1^2 + g_2^2}} & \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \\ 0 & \frac{g_2}{\sqrt{g_1^2 + g_2^2}} & \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \end{pmatrix} \begin{pmatrix} V_\mu^{(0)} \\ V_\mu^{(1)} \\ V_{3\mu}^{(2)} \end{pmatrix} \end{aligned} \quad (24)$$

where $i, j = 0, 1, 2$.

4 Axino Interactions

The axino interactions can be read off from the interaction lagrangian (9). The relevant term, written in terms of four components Majorana spinors⁶, is given by:

$$\mathcal{L} = i\sqrt{2}g_1^2 b_2^{(1)} \bar{\Lambda}^{(1)} \gamma_5 [\gamma^\mu, \gamma^\nu] (\partial_\mu V_\nu^{(1)}) \Psi_S + i\frac{\sqrt{2}}{2} g_2^2 b_2^{(2)} \bar{\Lambda}_3^{(2)} \gamma_5 [\gamma^\mu, \gamma^\nu] (\partial_\mu V_{3\nu}^{(2)}) \Psi_S \quad (25)$$

where the $b_2^{(a)}$ coefficients are given in (11). The related interaction vertex Feynman rule is

$$C^{(a)} [\gamma^\mu, \gamma^\nu] i k_\mu \quad (26)$$

where k_μ is the momentum of the outgoing vector and the $C^{(a)}$'s are

$$\begin{aligned} C^{(1)} &= \sqrt{2} g_1^2 b_2^{(1)} \\ C^{(2)} &= \frac{\sqrt{2}}{2} g_2^2 b_2^{(2)} \end{aligned} \quad (27)$$

The factor (27) contains the parameters $b_2^{(a)}$ which are related to the anomalous $U(1)$ (see eq. (11)). Therefore $C^{(a)} \ll g_a$ and the axino interactions will be extremely weak, being suppressed by an order of magnitude factor with respect to the weak interactions. At tree level there is only one type of annihilation diagram, represented in fig. 2. We

⁶The gamma matrices γ^μ are in the Weyl representation.

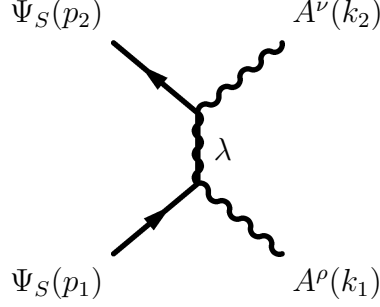


Figure 2: Annihilation of two axinos into two gauge vectors via the exchange of a gaugino.

denoted with p_1 and p_2 the incoming momenta of the axinos while k_1 and k_2 are the two outgoing momenta of the gauge bosons in the final state. We will concentrate on the case with two photons in the final state. In this case the result for the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{4M_S^2\omega_1}{16\pi^2(\omega_1 + \omega_2)^2(\sqrt{M_S^2 - E_2^2})} \sum_{i,j=1}^2 \mathcal{M}_i \mathcal{M}_j^* \quad (28)$$

where ω_1 and ω_2 are the energies of the two outgoing photons. Each amplitude \mathcal{M}_i is proportional to the related coefficient $C^{(a)}$ whose generic form is given in (27). The cross section (28), being extremely weak, cannot give a relic density in the WMAP preferred range. Thus we are forced to consider a scenario in which coannihilations between the axino and the NSLP became sizable. Several scenarios can be considered for the NLSP. We can split them into two major classes: one in which the NSLP is either a pure bino or a pure wino, and thus a coannihilation with a third MSSM particle is needed in order to recover the WMAP result, and one in which the NSLP is a generic MSSM neutralino with a non negligible bino and/or wino component. In both classes in order to have effective coannihilations the NSLP (and eventually the other MSSM particle involved in the coannihilation process) must be almost degenerate in mass. As a first example we consider a pure bino as the NLSP. The allowed coannihilation processes with the axino are those which involve an exchange of a photon or a Z_0 in the intermediate state and with a SM fermion-antifermion pair, Higgses and W 's in the final state. The diagram with the fermion-antifermion in the final state is sketched in fig. 3. The differential cross section in the center of mass frame has the following general form

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{s} \frac{p_f}{p_i} |\mathcal{M}|^2 \quad (29)$$

where s is the usual Mandelstam variable and $p_{f,i}$ is the spatial momentum of the outgoing (incoming) particles. On dimensional ground $|\mathcal{M}|^2$ has at least a linear dependence on p_f and this implies that the dominant contribution comes from the diagram with the SM

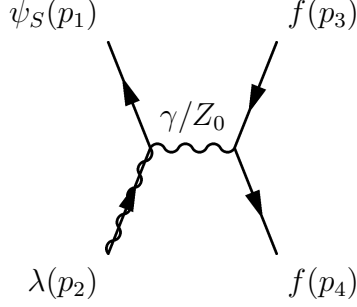


Figure 3: Coannihilation of an axino and a bino into a $f\bar{f}$ pair via the exchange of a photon or a Z_0 .

fermion-antifermion pair f and \bar{f} in the final state:

$$\Psi_S \lambda^{(a)} \rightarrow f \bar{f} \quad (30)$$

The resulting differential cross section, computed in the center of mass frame, is

$$\frac{d\sigma}{d\Omega} = \sum_f c_f \frac{\sqrt{(E_3 - m_f)^2}}{64\pi^2 (E_1 + E_2)^2 \sqrt{(E_1^2 - M_S^2)}} (\mathcal{M}_\gamma^2 + \mathcal{M}_{Z_0}^2 + \mathcal{M}_\gamma^* \mathcal{M}_{Z_0} + \mathcal{M}_\gamma \mathcal{M}_{Z_0}^*) \quad (31)$$

where the sum is extended to all the SM fermions (with mass m_f) while c_f is a color factor. Details of the amplitude computation can be found in Appendix A.

5 Axino Relic Density

In this section we compute the relic density of the axino. The case of the axino as a cold dark matter candidate has been studied for the first time in [18]. As we said in the previous section we study two scenarios: the first in which the axino coannihilates with only one NLSP degenerate in mass (a generic MSSM neutralino), the second in which there is an additional supersymmetric particle (either a chargino or a stau) involved in the coannihilation process with the axino and the NLSP.

Just to fix the notation we briefly review the relic density computation for N interacting species [19, 20, 21]. The Boltzmann equation for N particle species is given by:

$$\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^N \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{eq} n_j^{eq}) \quad (32)$$

where n_i denotes the number density per unit of comoving volume of the species $i = 1, \dots, N$ ($i = 1$ refers to the LSP, $i = 2$ refers to the NSLP, and so on), $n = \sum_i n_i$, H is the Hubble constant, σ_{ij} is the annihilation cross section between a species i and a species

j , v_{ij} is the modulus of the relative velocity while n_i^{eq} is the equilibrium number density of the species i :

$$n_i^{eq} = g_i (1 + \Delta_i)^{3/2} e^{-\Delta_i x_f} \quad (33)$$

where g_i are the internal degrees of freedom, $\Delta_i = (m_i - m_1)/m_1$ while $x_f = m_1/T$ is the freeze-out temperature. Numerical computation gives $x_f \simeq 20$ [20].

Eq. (32) can be rewritten in a useful way by defining the thermal average of the effective cross section

$$\langle \sigma_{eff} v \rangle \equiv \sum_{i,j=1}^N \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq} n_j^{eq}}{n^{eq} n^{eq}} \quad (34)$$

obtaining

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{eff} v \rangle (n^2 - (n^{eq})^2) \quad (35)$$

where $n^{eq} = \sum_i n_i^{eq}$. As a rule of thumb [22] a first order estimate of the relic density is given by

$$\Omega_\chi h^2 \simeq \frac{10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{eff} v \rangle} \quad (36)$$

We take into account the two cases $N = 2$ and $N = 3$:

- $N = 2$ case. Assuming that the relative velocities are all equal $v_{ij} \equiv v$ we get:

$$\langle \sigma_{eff}^{(2)} v \rangle = \langle \sigma_{22} v \rangle \frac{\langle \sigma_{11} v \rangle / \langle \sigma_{22} v \rangle + 2 \langle \sigma_{12} v \rangle / \langle \sigma_{22} v \rangle Q + Q^2}{(1 + Q)^2} \quad (37)$$

where $Q = n_2^{eq}/n_1^{eq}$. The first term in the numerator can be neglected because the axino annihilation cross section is suppressed by a factor $(C^{(a)})^4$ with respect to the MSSM neutralino annihilations (see the previous section) and thus $\langle \sigma_{11} v \rangle \ll \langle \sigma_{22} v \rangle$. The second term involves the coannihilation cross section. Let us consider the case in which the NLSP is a generic MSSM neutralino (a linear combination of $\lambda^{(1)}$, $\lambda_3^{(2)}$, \tilde{h}_d^0 , \tilde{h}_u^0) with a non vanishing bino or wino components. As we saw in the previous section each amplitude is generically proportional to $C^{(a)} g_i$ with $i = 1, 2$. Without loss of generality we consider the diagram which involves the bino component $\Psi_S \lambda^{(1)} \rightarrow f \bar{f}$ and a photon exchange in the intermediate channel, i.e the \mathcal{M}_γ^2 amplitude in (31). We get

$$C_\gamma^2 = (C^{(1)} \cos \theta_W)^2 = 2(b_2^{(1)})^2 g_1^4 \cos^2 \theta_W \quad (38)$$

From the expression of the mixed $U(1)' - U(1)_Y - U(1)_Y$ anomaly (see [7]) and from the eq. (11) we have the following relation

$$b_2^{(1)} = \frac{3(3Q_Q + Q_L)}{256\pi^2 b_3} \quad (39)$$

where $b_3 = M_{Z'}/4g_0$. With the assumption $M_{Z'} = 1$ TeV as in [7] we finally get

$$\frac{C_\gamma^2}{e^2} \simeq 5.76 \times 10^{-12} (3g_0 Q_Q + g_0 Q_L)^2 \text{ GeV}^{-2} \quad (40)$$

where e is the electric charge. We get similar expressions for the other three terms in (31). This result has to be compared to the typical weak cross section $\langle \sigma_{22} v \rangle \simeq 10^{-9} \text{ GeV}^{-2}$. As long as the charges and the coupling constant of the extra $U(1)$ satisfy the perturbative requirement

$$g_0^2 \cdot (3Q_Q + Q_L)^2 < 16 \quad (41)$$

the following upper bound is satisfied:

$$\frac{\langle \sigma_{12} v \rangle}{\langle \sigma_{22} v \rangle} \lesssim 10^{-6} \quad (42)$$

in the case of a pure bino, while

$$\frac{\langle \sigma_{12} v \rangle}{\langle \sigma_{22} v \rangle} \lesssim 10^{-5} \quad (43)$$

in the case of a pure wino. The previous ratios can be arbitrarily small, thus we can not derive a lower bound, but they cannot vanish since this would correspond to a complete decoupling of the axino. In this case the lightest MSSM state is the LSP. Accordingly to eqs. (36), (37), (42) and (43) the relic density gets rescaled as [23]

$$(\Omega h^2)^{(2)} \simeq \left[\frac{1+Q}{Q} \right]^2 (\Omega h^2)^{(1)} \quad (44)$$

We performed a random sampling of MSSM models in which the NLSP is a pure bino or a mixed bino-higgsino (the case of a pure wino falls back into the $N = 3$ case due to the wino-chargino mass degeneracy) and we computed the relic density in presence of coannihilations using the DarkSUSY package [24]. These two situations are easily realized in some corners of the mSUGRA parameter space. Thus in our scan we assumed this scenario in order to fix the pattern of the supersymmetry breaking parameters at weak scale. We emphasize here that this choice is completely arbitrary, and it is assumed only for simplicity, since in our model [7] the supersymmetry breaking mechanism is not specified. In the former case there is no model which satisfies the WMAP constraints [25]:

$$0.0913 \leq \Omega h^2 \leq 0.1285 \quad (45)$$

since the annihilation cross section of a pure bino is too low and the rescaling (44) is not enough to get the right relic density. In the latter case the higgsino component

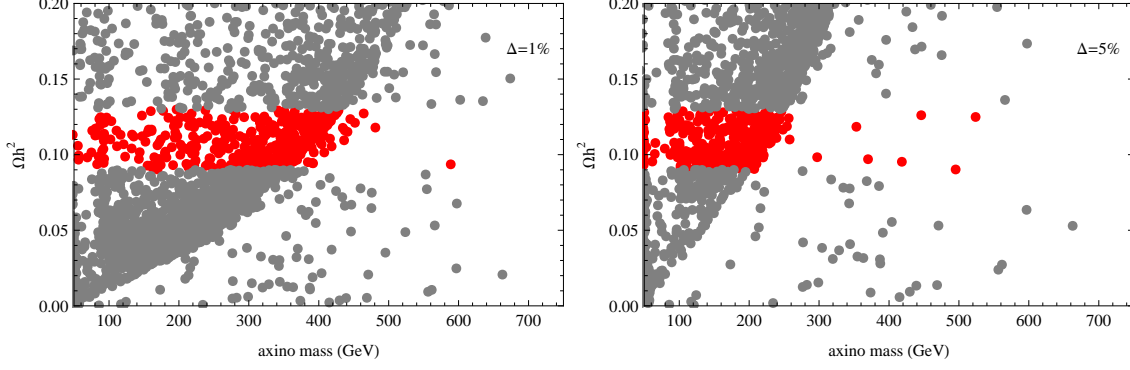


Figure 4: Axino relic density in the case in which the NSLP is a linear combination bino-higgsino. Red (darker) points denote models which satisfy WMAP data. Left panel: $\Delta_2 = 1\%$. Right panel: $\Delta_2 = 5\%$.

tends to increase the annihilation cross section and thus we find models which satisfy the WMAP constraints. The results are summarized in the fig. 4 for $\Delta_2 = 1\%$ and $\Delta_2 = 5\%$. In order to fulfill the WMAP data (red (darker) points in the plot (4)) the axino mass must be in the range $50 \text{ GeV} \lesssim M_S \lesssim 700 \text{ GeV}$ in the limit $\Delta_2 \rightarrow 0$, where the lowest bound is given by the current experimental constraints [26].

- $N = 3$ case. This is the case in which there is a third MSSM particle almost degenerate in mass with the LSP and the NLSP. Typical situations of this kind arise when the NLSP and the next to next to lightest supersymmetric particle (NNLSP) are respectively the bino and the stau or the wino and the lightest chargino. Expanding in an explicit way all the terms in the sum (34) we get:

$$\begin{aligned}
\langle \sigma_{\text{eff}}^{(3)} v \rangle &= \langle \sigma_{11} v \rangle \gamma_1^2 + \langle \sigma_{12} v \rangle \gamma_1 \gamma_2 + \langle \sigma_{13} v \rangle \gamma_1 \gamma_3 + \\
&\quad \langle \sigma_{21} v \rangle \gamma_2 \gamma_1 + \langle \sigma_{22} v \rangle \gamma_2^2 + \langle \sigma_{23} v \rangle \gamma_2 \gamma_3 + \\
&\quad \langle \sigma_{31} v \rangle \gamma_3 \gamma_1 + \langle \sigma_{32} v \rangle \gamma_3 \gamma_2 + \langle \sigma_{33} v \rangle \gamma_3^2 \\
&= \left[\langle \sigma_{11} v \rangle (n_1^{\text{eq}})^2 + \langle \sigma_{12} v \rangle n_1^{\text{eq}} n_2^{\text{eq}} + \langle \sigma_{13} v \rangle n_1^{\text{eq}} n_3^{\text{eq}} + \right. \\
&\quad \langle \sigma_{21} v \rangle n_2^{\text{eq}} n_1^{\text{eq}} + \langle \sigma_{22} v \rangle (n_2^{\text{eq}})^2 + \langle \sigma_{23} v \rangle n_2^{\text{eq}} n_3^{\text{eq}} + \\
&\quad \left. \langle \sigma_{31} v \rangle n_3^{\text{eq}} n_1^{\text{eq}} + \langle \sigma_{32} v \rangle n_3^{\text{eq}} n_2^{\text{eq}} + \langle \sigma_{33} v \rangle (n_3^{\text{eq}})^2 \right] \frac{1}{(n^{\text{eq}})^2} \\
&\simeq \frac{[\langle \sigma_{22} v \rangle (n_2^{\text{eq}})^2 + 2\langle \sigma_{23} v \rangle n_2^{\text{eq}} n_3^{\text{eq}} + \langle \sigma_{33} v \rangle (n_3^{\text{eq}})^2]}{(n^{\text{eq}})^2} \tag{46}
\end{aligned}$$

where in the last line we have neglected the terms $\langle \sigma_{11} v \rangle$, $\langle \sigma_{12} v \rangle$ and $\langle \sigma_{13} v \rangle$ since these are the thermal averaged cross sections which involve the axino. By introducing a new set of variables defined by

$$Q_i = \frac{n_i^{\text{eq}}}{n_1^{\text{eq}}} = \frac{g_i}{g_1} (1 + \Delta_i)^{3/2} e^{-x_f \Delta_i} \quad \text{for } i = 2, 3 \tag{47}$$

where g_i are the internal degrees of freedom of the particle species, $x_f = m_1/T$ and $\Delta_i = (m_i - m_1)/m_1$, we obtain

$$\langle \sigma_{\text{eff}}^{(3)} v \rangle \simeq \frac{\langle \sigma_{22} v \rangle Q_2^2 + 2\langle \sigma_{23} v \rangle Q_2 Q_3 + \langle \sigma_{33} v \rangle Q_3^2}{(1 + Q_2 + Q_3)^2} \quad (48)$$

Under the assumption $(m_3 - m_2)/m_1 \ll 1/x_f$, $Q_3/Q_2 \simeq g_3/g_2$ we finally get

$$\langle \sigma_{\text{eff}}^{(3)} v \rangle \simeq \frac{Q_2^2}{\left[1 + \left(1 + \frac{g_3}{g_2}\right) Q_2\right]^2} \langle \sigma_{\text{MSSM}} v \rangle \quad (49)$$

where

$$\langle \sigma_{\text{MSSM}} v \rangle = \langle \sigma_{22} v \rangle + 2\frac{g_3}{g_2} \langle \sigma_{23} v \rangle + \left(\frac{g_3}{g_2}\right)^2 \langle \sigma_{33} v \rangle \quad (50)$$

In order to compute the rescaling factor between the relic density of our model and the MSSM relic density we have to express σ_{MSSM} in terms of a two coannihilating species effective cross section. This is given by

$$\begin{aligned} \langle \sigma_{\text{eff}}^{(2)} v \rangle &= \frac{\langle \sigma_{22} v \rangle (n_2^{\text{eq}})^2 + 2\langle \sigma_{23} v \rangle n_2^{\text{eq}} n_3^{\text{eq}} + \langle \sigma_{33} v \rangle (n_3^{\text{eq}})^2}{(n^{\text{eq}})^2} \\ &= \frac{\langle \sigma_{22} v \rangle (n_2^{\text{eq}})^2 + 2\langle \sigma_{23} v \rangle n_2^{\text{eq}} n_3^{\text{eq}} + \langle \sigma_{33} v \rangle (n_3^{\text{eq}})^2}{(n_2^{\text{eq}} + n_3^{\text{eq}})^2} \\ &= \frac{\langle \sigma_{22} v \rangle (n_2^{\text{eq}})^2 + 2\langle \sigma_{23} v \rangle n_2^{\text{eq}} n_3^{\text{eq}} + \langle \sigma_{33} v \rangle (n_3^{\text{eq}})^2}{(n_2^{\text{eq}})^2 (1 + n_3^{\text{eq}}/n_2^{\text{eq}})^2} \\ &= \frac{\langle \sigma_{22} v \rangle + 2\langle \sigma_{23} v \rangle Q_{23} + \langle \sigma_{33} v \rangle Q_{23}^2}{(1 + Q_{23})^2} \end{aligned} \quad (51)$$

where

$$\begin{aligned} Q_{23} &= n_3^{\text{eq}}/n_2^{\text{eq}} = \frac{g_3}{g_2} \left(1 + \frac{m_3 - m_2}{m_2}\right)^{3/2} e^{-x_f \frac{m_3 - m_2}{m_2}} \\ &\simeq \frac{g_3}{g_2} \end{aligned} \quad (52)$$

since $(m_3 - m_2)/m_1 \ll 1/x_f$ and $m_2 > m_1$ then $(m_3 - m_2)/m_2 \ll 1/x_f$. We remind the reader that the values of n_2^{eq} , n_3^{eq} and n^{eq} are different with respect to those in the former case since now there are only two species in the thermal bath. We then find

$$\begin{aligned} \langle \sigma_{\text{eff}}^{(2)} v \rangle &\simeq \frac{\langle \sigma_{22} v \rangle + 2\frac{g_3}{g_2} \langle \sigma_{23} v \rangle + \left(\frac{g_3}{g_2}\right)^2 \langle \sigma_{33} v \rangle}{\left(1 + \frac{g_3}{g_2}\right)^2} \\ &\simeq \frac{\langle \sigma_{\text{MSSM}} v \rangle}{\left(1 + \frac{g_3}{g_2}\right)^2} \end{aligned} \quad (53)$$

and inserting back this relation into (49) we obtain

$$\langle \sigma_{\text{eff}}^{(3)} v \rangle \simeq \left[\frac{\left(1 + \frac{g_3}{g_2}\right) Q_2}{1 + \left(1 + \frac{g_3}{g_2}\right) Q_2} \right]^2 \langle \sigma_{\text{eff}}^{(2)} v \rangle \quad (54)$$

The rescaling factor between the three and two particle species relic density is given by the following relation

$$(\Omega h^2)^{(3)} \simeq \left[\frac{1 + \left(1 + \frac{g_3}{g_2}\right) Q_2}{\left(1 + \frac{g_3}{g_2}\right) Q_2} \right]^2 (\Omega h^2)^{(2)} \quad (55)$$

We performed a random sampling of MSSM models with bino-stau and wino-chargino coannihilations. The first situation is realized in some corners of the mSUGRA parameter space⁷ while the second situation is naturally realized in anomaly mediated supersymmetry breaking scenarios. For each model we computed the relic density $(\Omega h^2)^{(2)}$ for the two coannihilating species with the DarkSUSY package [24]. We finally computed $(\Omega h^2)^{(3)}$ using (55). The bino-stau models which satisfy the WMAP constraints have an axino mass in the range $100 \text{ GeV} \lesssim M_S \lesssim 350 \text{ GeV}$ in the limit $\Delta_2 \rightarrow 0$. As the mass gap increases the number of allowed models drastically decreases and eventually vanishes for $\Delta \simeq 5\%$. In the wino-chargino case, models which satisfy the WMAP constraints are shown in fig. 5 for four reference values of Δ_2 . The space of parameters with $\Delta_2 \lesssim 5\%$ and an axino mass $M_S \gtrsim 700 \text{ GeV}$ is favored while as the mass gap increases lower axino masses become favored, e.g. $100 \text{ GeV} \lesssim M_S < 200 \text{ GeV}$ ($\Delta_2 \simeq 20\%$).

6 Conclusions

We studied a possible dark matter candidate in the framework of our minimal anomalous $U(1)'$ extension of the MSSM [7]. In the decoupling limit (13) and under the assumption $M_0 \gg M_S, M_{V(0)}$ the axino turns out to be the LSP. Being an XWIMP the axino annihilation cross section is suppressed with respect to the typical weak interaction cross sections. This implies that in order to satisfy the WMAP constraints on the relic density we must have at least a NLSP almost degenerate in mass with the axino. We considered the case with two and three coannihilating particles and we found some configuration which satisfies the WMAP constraints. The results depend on the mass gap between the axino and the NLSP. In the exact degeneracy limit $\Delta_2 \rightarrow 0$ the allowed models have an axino mass in the range $50 \text{ GeV} \lesssim M_S \lesssim 700 \text{ GeV}$ for the bino-higgsino coannihilation case

⁷or in the so called Constrained MSSM (CMSSM).

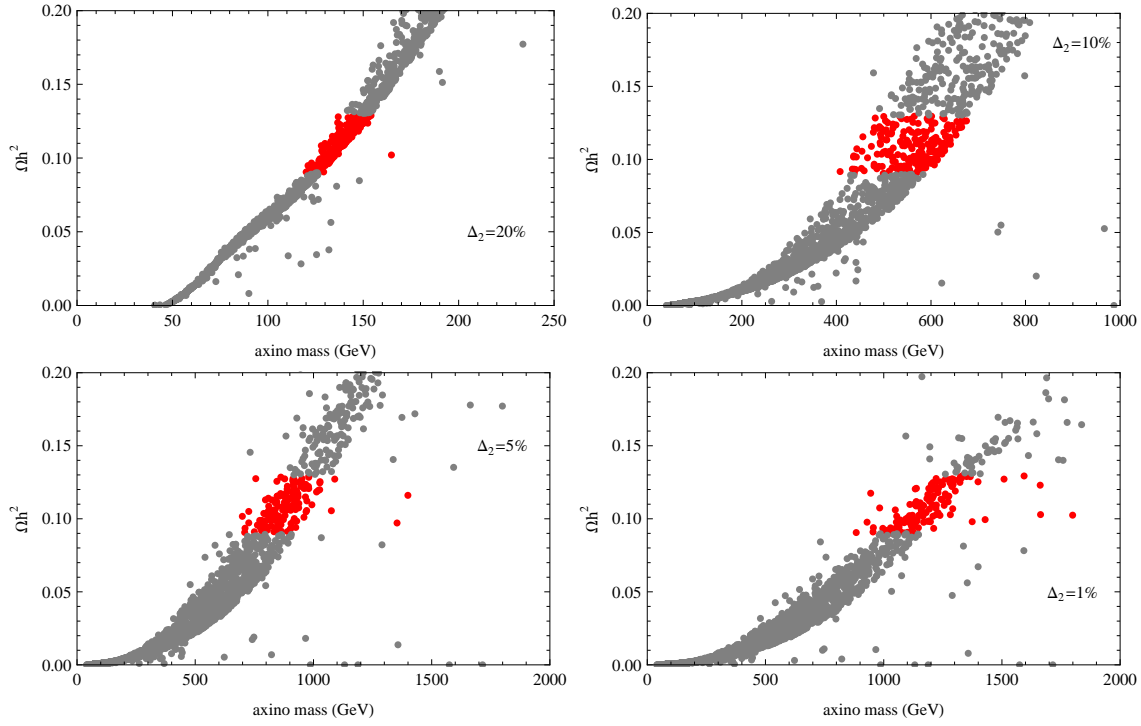


Figure 5: Axino relic density in the case in which the NSLP is a wino while the NNLSP is the lightest chargino. Red (darker) points denote models which satisfy WMAP data. Upper left panel: $\Delta_2 = 20\%$. Upper right panel: $\Delta_2 = 10\%$. Lower left panel: $\Delta_2 = 5\%$. Lower right panel: $\Delta_2 = 1\%$.

while $900 \text{ GeV} \lesssim M_S \lesssim 2 \text{ TeV}$ for the wino-chargino coannihilation case. When the mass gap is $\Delta_2 \simeq 20\%$ the allowed models are those with wino-chargino coannihilations and an axino mass of $100 \text{ GeV} \lesssim M_S < 200 \text{ GeV}$. Finally let us comment on the differences between our scenario and that studied in the work [23]. In our framework the $U(1)'$ does not arise from a hidden sector and thus all the MSSM fields can be charged under this extra abelian gauge group. This is the most relevant feature which could also be detected experimentally (see for example [6]). Moreover in our scenario the axino interactions are suppressed with respect to the weak interactions due to the GS couplings while in [23] the mechanism to suppress the couplings and give an XWIMP is provided by the kinetic mixing between the $U(1)'$ and $U(1)_Y$.

A Amplitude for $\lambda_1 + \psi_S \rightarrow f \bar{f}$

In this Appendix we give some detail about the amplitude computation for the process $\lambda_1 + \psi_S \rightarrow f \bar{f}$,

$$\mathcal{M} = -ik^\mu \bar{v}_S \gamma_5 [\gamma_\mu, \gamma_\nu] u_1 \left[eq_f C_\gamma \frac{\eta^{\nu\rho}}{k^2} \bar{u}_f \gamma_\rho v_f + \frac{g_{Z_0}}{2} C_{Z_0} \frac{\eta^{\nu\rho}}{k^2 - M_{Z_0}^2} \bar{u}_f \gamma_\rho (v_f^{Z_0} - a_f^{Z_0} \gamma_5) v_f \right] \quad (56)$$

where $C_\gamma = C^{(1)} \cos \theta_W$, $C_{Z_0} = -C^{(1)} \sin \theta_W$ while $k^2 = s$ is the momentum of the intermediate gauge boson. The corresponding square modulus is

$$|\mathcal{M}|^2 = -64 \left[T_a \left(\frac{a_f C_{Z_0} g_{Z_0}}{k^2 - M_{Z_0}^2} \right)^2 + T_v \left(\frac{2C_\gamma eq_f}{k^2} + \frac{C_{Z_0} g_{Z_0} v_f}{k^2 - M_{Z_0}^2} \right)^2 \right] \quad (57)$$

with

$$\begin{aligned} T_v &= m_f^4 (p_{\lambda_1} p_S) + M_1 M_S \left[2m_f^4 + 3(p_f p_{\bar{f}}) m_f^2 + (p_f p_{\bar{f}})^2 \right] + \\ &\quad - (p_f p_{\bar{f}}) \left[(p_{\lambda_1} p_f)(p_f p_S) + (p_{\lambda_1} p_{\bar{f}})(p_{\bar{f}} p_S) \right] + m_f^2 \left[(p_{\lambda_1} p_S)(p_f p_{\bar{f}}) + \right. \\ &\quad \left. - 2(p_{\lambda_1} p_f)(p_f p_S) - (p_{\lambda_1} p_{\bar{f}})(p_f p_S) - (p_{\lambda_1} p_f)(p_{\bar{f}} p_S) - 2(p_{\lambda_1} p_{\bar{f}})(p_{\bar{f}} p_S) \right] \\ T_a &= \left[(p_{\lambda_1} p_{\bar{f}})(p_f p_S) + (p_{\lambda_1} p_f)(p_{\bar{f}} p_S) \right] m_f^2 - M_1 M_S \left[m_f^4 - (p_f p_{\bar{f}})^2 \right] + \\ &\quad - (p_f p_{\bar{f}}) \left[(p_{\lambda_1} p_f)(p_f p_S) + (p_{\lambda_1} p_{\bar{f}})(p_{\bar{f}} p_S) \right] \end{aligned} \quad (58)$$

where p_{λ_1} , p_S , p_f and $p_{\bar{f}}$ are the bino, axino and SM fermions 4-momenta respectively. Writing all the momenta in function of s and integrating over the solid angle we get

$$\begin{aligned} \sigma &= c_f \left(g_1^2 b_2^{(1)} \right)^2 \sqrt{s - 4m_f^2} \times \\ &\quad \times \frac{\left[-2M_1^4 + (4M_S^2 + s) M_1^2 - 6M_S s M_1 - 2M_S^4 + s^2 + M_S^2 s \right]}{12\pi (M_{Z_0}^2 - s)^2 s^{5/2} \sqrt{M_1^4 - 2(M_S^2 + s) M_1^2 + (M_S^2 - s)^2}} \times \\ &\quad \times \left[(2m_f^2 + s) \left(2 \cos \theta_W eq_f (M_{Z_0}^2 - s) + \sin \theta_W g_{Z_0} v_f s \right)^2 + (\sin \theta_W g_{Z_0} a_f)^2 s^2 (s - 4m_f^2) \right] \end{aligned} \quad (59)$$

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